

Math 1A**Midterm 2 Review Answers**

$$[1] \quad \csc 0.5 \approx \csc \frac{\pi}{6} + \left(-\csc \frac{\pi}{6} \cot \frac{\pi}{6} \right) \left(0.5 - \frac{\pi}{6} \right) = \frac{6 + (\pi - 3)\sqrt{3}}{3}$$

$$[2] \quad dx = \Delta x = 0.5$$

$$\Delta y = y(2 + 0.5) - y(2) = \frac{4}{25} - \frac{1}{4} = -\frac{9}{100}$$

$$dy = \left. \frac{dy}{dx} \right|_{x=2} \Delta x = -\frac{2}{(2)^3} (0.5) = -\frac{1}{8}$$

$$\begin{aligned}
 [3] \quad & \frac{d^3}{dx^3} \sec x \\
 &= \frac{d^2}{dx^2} \sec x \tan x \\
 &= \frac{d}{dx} (\sec x \tan^2 x + \sec^3 x) \\
 &= \sec x \tan^3 x + 2 \sec^3 x \tan x + 3 \sec^3 x \tan x \\
 &= \sec x \tan^3 x + 5 \sec^3 x \tan x
 \end{aligned}$$

$$[4] \quad s(t) = 2t^{\frac{5}{2}} + 4t^{\frac{3}{2}} - 3t^{-\frac{1}{2}}$$

$$[a] \quad s'(t) = 5t^{\frac{3}{2}} + 6t^{\frac{1}{2}} + \frac{3}{2}t^{-\frac{3}{2}} \Rightarrow s'(1) = \frac{25}{2}$$

$$[b] \quad s''(t) = \frac{15}{2}t^{\frac{1}{2}} + 3t^{-\frac{1}{2}} - \frac{9}{4}t^{-\frac{5}{2}} = \frac{3}{4}t^{-\frac{5}{2}}(10t^3 + 4t^2 - 3)$$

$$[5] \quad \text{The given line has slope } -\frac{1}{12}, \text{ so the tangent lines must have slope } 12$$

$$\frac{dy}{dx} = 3x^2 = 12 \Rightarrow x = \pm 2$$

$$\text{The tangent lines are } y - 9 = 12(x - 2) \text{ and } y + 7 = 12(x + 2)$$

$$[6] \quad \text{Since the function is quadratic, } f(x) = ax^2 + bx + c \text{ where } f(1) = -1, f'(1) = 3 \text{ and } f(2) = 4$$

$$\text{So, } a + b + c = -1, 2a + b = 3 \text{ and } 4a + 2b + c = 4$$

$$\text{Solving for the coefficients, } f(x) = 2x^2 - x - 2$$

$$\text{The tangent line is } y - 4 = 7(x - 2)$$

$$[7] \quad f'(x) = \frac{3x^2(1+x^2) - x^3(2x)}{(1+x^2)^2} = \frac{3x^2 + x^4}{(1+x^2)^2}$$

$$f''(x) = \frac{(6x + 4x^3)(1+x^2)^2 - (3x^2 + x^4)(2(1+x^2)(2x))}{(1+x^2)^4}$$

$$f''(1) = \frac{(6+4)(1+1)^2 - (3+1)(2(1+1)(2))}{(1+1)^4} = \frac{40-32}{16} = \frac{1}{2} \quad \text{NO NEED TO SIMPLIFY } f''(x) \text{ to find } f''(1)$$

[8] [a] $k(2) = 8f(2)$ and $k'(x) = 3x^2 f(x) + x^3 f'(x) \Rightarrow k'(2) = 12f(2) + 8f'(2)$

The tangent line is $y + 8 = 12(x - 2)$

[b] $j(-1) = \frac{1}{f(-1)}$ and $j'(x) = \frac{2xf(x) - x^2 f'(x)}{[f(x)]^2} \Rightarrow j'(-1) = \frac{-2f(-1) - f'(-1)}{[f(-1)]^2}$

The tangent line is $y - \frac{1}{2} = -\frac{1}{4}(x + 1)$

[c] $m(-3) = \tan^{-1}(g(-3))$ and $m'(x) = \frac{1}{1+[g(x)]^2} g'(x) \Rightarrow m'(-3) = \frac{1}{1+[g(-3)]^2} g'(-3)$

The tangent line is $y + \frac{\pi}{4} = x + 3$

[d] $n(4) = g(f(4))$ and $n'(x) = g'(f(x))f'(x) \Rightarrow n'(4) = g'(f(4))f'(4) = g'(3)f'(4)$

The tangent line is $y = -3(x - 4)$

[9] **ANSWER WITHHELD (SHOW ME YOUR SOLUTION FOR VERIFICATION)**

[10] $f'(x) = 4 - 3\sec^2 x = 0 \Rightarrow \sec x = \pm \frac{2}{\sqrt{3}} \Rightarrow \cos x = \pm \frac{\sqrt{3}}{2} \Rightarrow x = \frac{\pi}{6}, \frac{5\pi}{6}, \frac{7\pi}{6}, \frac{11\pi}{6}$

[11] $f'(x) = g(x^2) + xg'(x^2)(2x) = g(x^2) + 2x^2 g'(x^2)$
 $f''(x^2) = g'(x^2)(2x) + 4xg'(x^2) + 2x^2 g''(x^2)(2x) = 6xg'(x^2) + 4x^3 g''(x^2)$

[12] $5(1 + x^2 y^3)^4 (2xy^3 + 3x^2 y^2 \frac{dy}{dx}) = 4x^3 e^y + x^4 e^y \frac{dy}{dx}$
 $5(1 + 0)^4 (0 + 0) = 4(-1)(1) + (1)(1) \frac{dy}{dx} \Big|_{(-1, 0)} \Rightarrow \frac{dy}{dx} \Big|_{(-1, 0)} = 4$ **NO NEED TO SIMPLIFY $\frac{dy}{dx}$ to find $\frac{dy}{dx} \Big|_{(-1, 0)}$**
 $y = 4(x + 1)$

[13] $y = ax^4 \Rightarrow \frac{dy}{dx} = 4ax^3$
 $x^2 + 4y^2 = b^2 \Rightarrow 2x + 8y \frac{dy}{dx} = 0 \Rightarrow \frac{dy}{dx} = -\frac{x}{4y}$
 $4ax^3 \left(-\frac{x}{4y} \right) = -\frac{ax^4}{y} = -\frac{y}{y} = -1$

[14] $\ln y = \frac{\ln \sin x}{x}$
 $\frac{1}{y} \frac{dy}{dx} = \frac{\frac{1}{\sin x} (\cos x)x - \ln \sin x}{x^2} = \frac{x \cot x - \ln \sin x}{x^2}$
 $\frac{dy}{dx} = \frac{x \cot x - \ln \sin x}{x^2} (\sin x)^{\frac{1}{x}}$

[15] $f(x) = xe^{-x}$, $a = -1$ and $\lim_{h \rightarrow 0} \frac{(h-1)e^{1-h} + e}{h} = f'(-1) = (e^{-x} - xe^{-x}) \Big|_{x=-1} = 2e$